I. Quantum Revolution of Your Generation: Entangled States

We encountered two-entity states of the form

$$\left.\begin{array}{l} \text{2-spin: } \psi(1,2) \propto |\uparrow\rangle_1 |\downarrow\rangle_2 \pm |\downarrow\rangle_1 |\uparrow\rangle_2 \quad \text{OR} \quad \alpha(1)\beta(2) \pm \beta(1)\alpha(2) \\ \\ \text{2-particle: } \quad \phi_a(\vec{r}_1)\,\phi_b(\vec{r}_2) \pm \phi_b(\vec{r}_1)\,\phi_a(\vec{r}_2) \end{array}\right\} \quad (52)$$

These are entangled states,

first introduced by Schrödinger in 1935 (his cat!)

- According to QM, the 2-entity wavefunction contains all the information of the 2-entity system. Even so, it does not carry definite property for individual entity.

- The two entities (1 & 2) are strongly correlated

Forget about atoms for the moment

Some source

↓

two entities (spins) in

$$\psi(1,2) \sim \left(|\uparrow\rangle_1 |\downarrow\rangle_2 \pm |\downarrow\rangle_1 |\uparrow\rangle_2 \right) \text{ state}$$

and they fly apart

entity 1 ←   → entity 2

Entity 1 does <u>not</u> have a definite spin m_s (before measurement)
[in contrast to classical thinking]

Entity 2 does <u>not</u> have a definite spin (before measurement)

This quantum effect is there even the entities are far far apart

- Only when a measurement is done (say on entity 1), entity 1's spin becomes a reality (has a value), if it is $|\uparrow\rangle_1$ (up), then entity 2's spin must be $|\downarrow\rangle_2$; and vice versa.

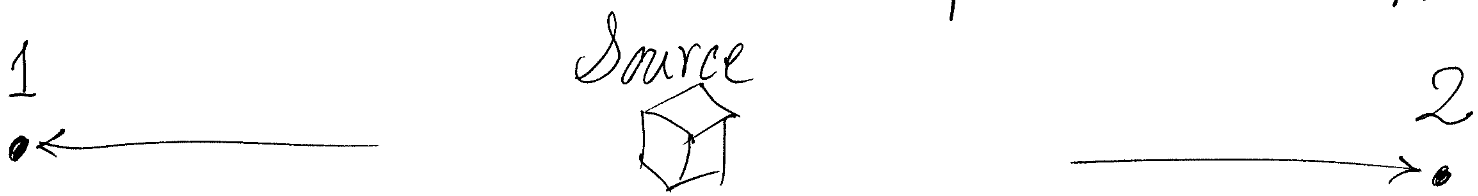
[i.e. one's reality affects the other, even they are far apart]

[measurement results are correlated in a way that classical physics can't explain!]

[states (52) have stronger-than-classical correlation]

$$\psi(1,2) \sim |\uparrow\rangle_1 |\downarrow\rangle_2 \pm |\downarrow\rangle_1 |\uparrow\rangle_2$$

[A measurement either picks up $|\uparrow\rangle_1 |\downarrow\rangle_2$ OR $|\downarrow\rangle_1 |\uparrow\rangle_2$]



if $|\uparrow\rangle_1$ — must be $|\downarrow\rangle_2$ (instantaneously)

if $|\downarrow\rangle_1$ — must be $|\uparrow\rangle_2$

[no matter how far 1 & 2 are]

Einstein, Podolsky, Rosen ("EPR") didn't like it!

"Spooky action at a distance"

How could measuring 1 affect state of 2 at no time? Is QM a complete theory?

Experiments since 1980's showed that entangled states behave in the way QM predicts. Entangled states are beyond the scope of Classical Physics.

Two properties for Second Quantum Revolution

$$\psi(1,2) \sim |\uparrow\rangle_1 |\downarrow\rangle_2 \pm |\downarrow\rangle_1 |\uparrow\rangle_2$$

- Superposition
- Entanglement (concerns two or more objects)

spins, electrons, ions, etc.

These two properties are essential for . . .

- Quantum computing
- Quantum information
- Quantum teleportation

⋮

- Experimental challenges: How to form $\psi(1,2)$ for two objects?
  How to keep $\psi(1,2)$ for sufficient time to do manipulations?

[Other QM-related courses and research work in Department]

Remarks

- Two objects, each spin-½ ["spin-up" or "spin-down"]

  $$|\uparrow\rangle_1 |\uparrow\rangle_2 , \quad |\uparrow\rangle_1 |\downarrow\rangle_2 , \quad |\downarrow\rangle_1 |\uparrow\rangle_2 , \quad |\downarrow\rangle_1 |\downarrow\rangle_2 \text{ span the space}$$

- We saw $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\downarrow\rangle_2 \pm |\downarrow\rangle_1 |\uparrow\rangle_2\right)$ are entangled states
- $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\uparrow\rangle_2 \pm |\downarrow\rangle_1 |\downarrow\rangle_2\right)$ are also entangled states
- These four are the maximally entangled states ("Bell states")
- Anything special about spin-½ (spins) particles?
  - Two-level atoms/ions —— ← there or —— ← here
  - Photons

- What so special about the Bell states?

Spin # 1 : $\phi_1 = a_1|\uparrow\rangle_1 + b_1|\downarrow\rangle_1$ (most general single spin-½ state)

Spin # 2 : $\phi_2 = a_2|\uparrow\rangle_2 + b_2|\downarrow\rangle_2$

If they are independent, then the 2-spin state will be

$$\phi_1 \cdot \phi_2 = (a_1|\uparrow\rangle_1 + b_1|\downarrow\rangle_1)(a_2|\uparrow\rangle_2 + b_2|\downarrow\rangle_2)$$

$$= a_1 a_2 |\uparrow\rangle_1|\uparrow\rangle_2 + a_1 b_2 |\uparrow\rangle_1|\downarrow\rangle_2 + b_1 a_2 |\downarrow\rangle_1|\uparrow\rangle_2 + b_1 b_2 |\downarrow\rangle_1|\downarrow\rangle_2$$

Q: For a Bell state, can you find the set of $(a_1, b_1; a_2, b_2)$?

[Hint: What does "Yes" imply? What does "No" imply?]

[This will be discussed in more advanced courses.]